Quintic Quasitopological Gravity

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Quasitopological Gravity

Context: AdS/CFT Correspondence

- Higher curvature theories have received some attention.
- Lovelock's Theory as a natural extension of General Relativity.
- The curvature invariants of $k-$th order in Lovelock theory don't contribute to the field equations if $D \leq 2k$.
- Contrary to this last property, new theories like Quasitopological Gravities appears.

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Quasitopological Gravity

- First result on 2010 [J Oliva, S. Ray: arxiv.org/1003.4773], introducing a cubic interaction in $D = 5$:

$$\mathcal{L}_3 = -\frac{7}{6} R^{ab}{}_{cd} R^{ce}{}_{bf} R^{df}{}_{ae} - R_{ab}{}^{cd} R_{cd}{}^{be} R^{a}{}_{e} - \frac{1}{2} R_{ab}{}^{cd} R^{a}{}_{c} R^{b}{}_{d} + \frac{1}{3} R^{a}{}_{b} R^{b}{}_{c} R^{c}{}_{a} - \frac{1}{2} R R^{a}{}_{b} R^{b}{}_{a} + \frac{1}{12} R^{3}. \quad (1)$$

- On spacetimes with spherical/planar/hyperbolic symmetry, the theory has second order field equations.
- Among others, this theory has the following properties:
  - The trace of the field equations is proportional to the Lagrangian.
  - Birkhoff's Theorem.
  - Interaction with GR and Gauss-Bonnet terms lead to an asymptotically AdS black hole.

Quasitopological Gravity

Goals of the speech:

- To present a theory in $D = 5$ which is fifth order in curvature, but with second order field equations on spherical/planar/hyperbolic spacetimes.
- To give some properties:
  - Birkhoff's Theorem.
  - No ghosts on AdS.

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The Theory

Here we are considering the following gravity theory:

$$I[g_{\mu\nu}] = \int d^5x\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi G} + \sum_{k=2}^{5} a_k \mathcal{L}_k\right] \qquad (2)$$

where $\mathcal{L}_2$ stands for the Gauss-Bonnet combination

$$\mathcal{L}_2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd},$$

$\mathcal{L}_3$ was defined on (1). The quartic quasitopological term $\mathcal{L}_4$ can be written as

$$\begin{aligned}\mathcal{L}_4 = \frac{1}{73\times 2^5 \times 3^2}\Big[&7080R^{pqbs}R_{p\ b}^{\ a\ u}R_{a\ u}^{\ v\ w}R_{qvsw} - 234R^{pqbs}R_{pq}^{\ \ au}R_{au}^{\ \ vw}R_{bsvw} - 1237\left(R^{pqbs}R_{pqbs}\right)^2\\ &+ 1216R^{pq}R^{bsau}R_{bs\ p}^{\ \ v}R_{auvq} - 6912R^{pq}R^{bs}R^{a\ u}_{\ p\ q}R_{abus} - 7152R^{pq}R^{bs}R^{au}_{\ \ pb}R_{auqs}\\ &+ 308R^{pq}R_{pq}R^{bsau}R_{bsau} + 298R^2R^{pqbs}R_{pqbs} + 12864R^{pq}R^{bs}R_{b}^{\ a}R_{psqa} - 115R^4\\ &-912RR^{pq}R^{bs}R_{pbqs} + 4112R^{pq}R_{p}^{\ b}R_{q}^{\ s}R_{bs} - 4256RR^{pq}R_{p}^{\ b}R_{qb} + 1156R^2R^{pq}R_{pq}\Big]\end{aligned}$$

The Theory

The new quintic Quasitopological combination is

$$
\begin{aligned}
\mathcal{L}_5 = {} & A_1 R R_b{}^a R_c{}^b R_d{}^c R_a{}^d + A_2 R R_b{}^a R_a{}^b R_{ef}{}^{cd} R_{cd}{}^{ef} + A_3 R R_c{}^a R_d{}^b R_{ef}{}^{cd} R_{ab}{}^{ef} \\
& + A_4 R_b{}^a R_a{}^b R_d{}^c R_e{}^d R_c{}^e + A_5 R_b{}^a R_c{}^b R_a{}^c R_{fg}{}^{de} R_{de}{}^{fg} + A_6 R_b{}^a R_d{}^b R_f{}^c R_{ag}{}^{de} R_{ce}{}^{fg} \\
& + A_7 R_b{}^a R_d{}^b R_f{}^c R_{cg}{}^{de} R_{ae}{}^{fg} + A_8 R_b{}^a R_c{}^b R_{ae}{}^{cd} R_{gh}{}^{ef} R_{df}{}^{gh} + A_9 R_b{}^a R_c{}^b R_{ef}{}^{cd} R_{gh}{}^{ef} R_{ad}{}^{gh} \\
& + A_{10} R_b{}^a R_c{}^b R_{eg}{}^{cd} R_{ah}{}^{ef} R_{df}{}^{gh} + A_{11} R_c{}^a R_d{}^b R_{ab}{}^{cd} R_{gh}{}^{ef} R_{ef}{}^{gh} + A_{12} R_c{}^a R_d{}^b R_{ae}{}^{cd} R_{gh}{}^{ef} R_{bf}{}^{gh} \\
& + A_{13} R_c{}^a R_d{}^b R_{ef}{}^{cd} R_{gh}{}^{ef} R_{ab}{}^{gh} + A_{14} R_c{}^a R_d{}^b R_{eg}{}^{cd} R_{ah}{}^{ef} R_{bf}{}^{gh} + A_{15} R_c{}^a R_e{}^b R_{af}{}^{cd} R_{gh}{}^{ef} R_{bd}{}^{gh} \\
& + A_{16} R_b{}^a R_{ad}{}^{bc} R_{fh}{}^{de} R_{ci}{}^{fg} R_{eg}{}^{hi} + A_{17} R_b{}^a R_{de}{}^{bc} R_{cf}{}^{de} R_{hi}{}^{fg} R_{ag}{}^{hi} + A_{18} R_b{}^a R_{df}{}^{bc} R_{ac}{}^{de} R_{hi}{}^{fg} R_{eg}{}^{hi} \\
& + A_{19} R_b{}^a R_{df}{}^{bc} R_{ah}{}^{de} R_{ei}{}^{fg} R_{cg}{}^{hi} + A_{20} R_b{}^a R_{df}{}^{bc} R_{gh}{}^{de} R_{ei}{}^{fg} R_{ac}{}^{hi} + A_{21} R_{cd}{}^{ab} R_{eg}{}^{cd} R_{ai}{}^{ef} R_{fj}{}^{gh} R_{bh}{}^{ij} \\
& + A_{22} R_{ce}{}^{ab} R_{af}{}^{cd} R_{gi}{}^{ef} R_{bj}{}^{gh} R_{dh}{}^{ij} + A_{23} R_{ce}{}^{ab} R_{ag}{}^{cd} R_{bi}{}^{ef} R_{fj}{}^{gh} R_{dh}{}^{ij} + A_{24} R_{ce}{}^{ab} R_{fg}{}^{cd} R_{hi}{}^{ef} R_{aj}{}^{gh} R_{bd}{}^{ij} \,, \qquad (3)
\end{aligned}
$$

The Theory

And the coefficients that define the new quintic quasi-topological interaction in (3) are:

$$
\begin{aligned}
&A_1 = \frac{9497}{17767320}, \quad A_2 = -\frac{759299}{71069280}, \quad A_3 = \frac{124967}{5922440}, \quad A_4 = \frac{759299}{23689760},\\
&A_5 = \frac{197761}{11844880}, \quad A_6 = \frac{1362599}{23689760}, \quad A_7 = -\frac{5006573}{11844880}, \quad A_8 = -\frac{9290347}{71069280},\\
&A_9 = \frac{3400579}{11844880}, \quad A_{10} = -\frac{6726521}{11844880}, \quad A_{11} = \frac{363777}{23689760}, \quad A_{12} = \frac{6348187}{47379520},\\
&A_{13} = -\frac{9487667}{71069280}, \quad A_{14} = -\frac{6454201}{8883660}, \quad A_{15} = -\frac{34697591}{142138560}, \quad A_{16} = -\frac{5643853}{71069280},\\
&A_{17} = -\frac{29094011}{71069280}, \quad A_{18} = -\frac{48458099}{71069280}, \quad A_{19} = \frac{1547591}{740305}, \quad A_{20} = \frac{78763919}{71069280},\\
&A_{21} = -\frac{10718341}{17767320}, \quad A_{22} = \frac{9629717}{17767320}, \quad A_{23} = \frac{1113473}{17767320}, \quad A_{24} = -\frac{16111757}{17767320}.
\end{aligned}
$$

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The Theory

- The quasitopological gravities are defined up to the addition of the corresponding Euler densities.
- A geometric interpretation of the quasitopological theories remains as an open problem.

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Some Properties: Birkhoff's Theorem

Claim: For generic values of the couplings a_k, the spherically (planar or hyperbolic) symmetric solution is static and it is determined by a quintic polynomial equation.

- The proof is done through the Reduced Action approach, evaluating the Lagrangian on the metric

$$ds^2 = -f(t,r)b^2(t,r)dt^2 + 2m(t,r)b(t,r)\, dtdr + \frac{dr^2}{f(t,r)} + r^2 d\Sigma_\gamma^2.$$

Here $d\Sigma_\gamma$ denotes the line element of a Euclidean 3d manifold of constant curvature $\gamma \in \{\pm 1, 0\}$.

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Some Properties: Birkhoff's Theorem

- For convenience, we define $h(t,r) = f(t,r) - \gamma$. The variation of the reduced action with respect to h, b, m, and a posteriori gauge fixing $m(t,r) = 0$ leads to:

$$0 = (-24r^6 h(t,r)a_2 - 6r^4 h(t,r)^2 a_3 - 4r^2 h(t,r)^3 a_4 + 5h(t,r)^4 a_5 + 6r^8)\frac{\partial b(t,r)}{\partial r} \quad (4)$$

$$0 = h(t,r)^5 r^{-5} a_5 - h(t,r)^4 r^{-3} a_4 - 2h(t,r)^3 r^{-1} a_3 - 12h(t,r)^2 r a_2 + r^5\Lambda + 6r^3 h(t,r) + \mu(t) r \quad (5)$$

$$0 = (-24r^6 h(t,r)a_2 - 6r^4 h(t,r)^2 a_3 - 4r^2 h(t,r)^3 a_4 + 5h(t,r)^4 a_5 + 6r^8)\frac{\partial h(t,r)}{\partial t} \quad (6)$$

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Some Properties: Birkhoff's Theorem

- The fact that the values a_k are generic induces that $\frac{\partial h(t,r)}{\partial t} = \frac{\partial b(t,r)}{\partial r} = 0$, hence $\mu(t)$ is in fact constant.
- From this, $b(t)$ can be absorbed by a time reparametrization, which means that the metric now reads:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_\gamma^2,$$

Now it's easy to see that the solution is static.

Some Properties: No-ghosts around AdS

Claim: Around maximally symmetric backgrounds, quintic quasitopological gravities lead to the same propagator that G.R, with an effective Newton's constant which depends on the values of the couplings a_k.

- Fast-linearization procedure for gravity theories involving contractions of the Riemman tensor, ie, $\mathcal{L}(R_{\alpha\beta\rho\sigma}, g_{\mu\nu})$ around maximally symmetric backgrounds.
  [P. Bueno, P.Cano: arxiv.org/1607.06463].

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Some Properties: No-ghosts around AdS

- The linearized field equations are written in terms of values a, b, c, e, which depend on the theory under consideration.
- Their method consists in the evaluation of the Lagrangian on a deformed curvature that depends on two parameters, (α, χ).
- The values a, b, c, e can be obtained by taking specific derivatives and evaluations on this effective action.

Some Properties: No-ghosts around AdS

- In the quintic quasitopological gravity case, we assume that the maximally symmetric solution has a dressed constant curvature, λ, which is fixed by the polynomial

$$P[\lambda] := a_5\lambda^5 + 6a_4\lambda^4 - 72a_3\lambda^3 + 2592a_2\lambda^2 + 7776(\Lambda - \lambda) = 0$$

- Scaling $\lambda \to \frac{\lambda}{6}$ for simplicity, the linearized equations read

$$\frac{dP[\lambda]}{d\lambda} G^L_{\mu\nu} = 0,$$

where $G^L_{\mu\nu}$ is the linearized Einstein tensor.

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Final Comments

- Assuming that (5) has a solution $f(r)$ with a single zero located at $r = r_h$, the Black Hole thermodynamical properties can be analyzed. In this case we have:

$$T = \frac{1}{2\pi r_h}\left(\frac{-3a_5\gamma^5 - 2a_4\gamma^4 r_h^2 + 2a_3\gamma^3 r_h^4 + 6\gamma r_h^8 - 2\Lambda r_h^{10}}{5a_5\gamma^4 + 4a_4\gamma^3 r_h^2 - 6a_3\gamma^2 r_h^4 + 24a_2\gamma r_h^6 + 6r_h^8}\right)$$

$$\mathcal{S} = Vol(\Sigma_\gamma)\left(4\pi r_h^3 + 48\pi\gamma a_2 r_h + \frac{12\pi\gamma^2 a_3}{r_h^2} - \frac{8\pi\gamma^3 a_4}{3r_h^3} - \frac{2\pi\gamma^4 a_5}{r_h^5}\right)$$

$$\mathcal{M} = \frac{Vol(\Sigma_\gamma)}{2}\left(\frac{\gamma^5 a_5}{r_h^6} + \frac{\gamma^4 a_4}{r_h^4} - \frac{2\gamma a_3}{r_h^2} + 12\gamma^2 a_2 + 6\gamma r_h^2 - \Lambda r_h^4\right)$$

satisfying the 1st Law of Thermodynamics, $d\mathcal{M} = T\, d\mathcal{S}$.

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Final Comments

Quasitopological Gravities shares with its Lovelock counterpart a lot of properties. For mention a few:

- 2nd order field equations, although QTG have shown to have this property on spherically/planar/hyperbolic spacetimes.
- The asymptotic behavior allowed by Wheeler's polynomial coincides with that of General Relativity.
- Birkhoff's Theorem.

Final Comments

THANKS FOR YOUR ATTENTION!