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Inner Approximations of Probability Functions via the Moreau Envelope.

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Abstract

Optimization problems with uncertainty in the constraints occur in many applications. Particularly, probability functions present a natural form to deal with this situation. Nevertheless, in some cases, the resulting probability functions are nonsmooth. This motivates us to propose a regularization employing the Moreau envelope of a scalar representation of the vector inequality. More precisely, we consider a probability function which covers most of the general classes of probabilistic constraints:

$$\varphi(x) = \mathbb{P}(\Phi(x, \xi) \in -\mathcal{K}),$$

where $\mathcal{K}$ is a convex cone of a Banach space. The conic inclusion $\Phi(x, \xi) \in -\mathcal{K}$ represents an abstract system of inequalities, and ξ is a random vector. We propose a regularization by applying the Moreau envelope to the scalarization of the function Φ. In this talk, we demonstrate, under mild assumptions, the smoothness of such a regularization and that it satisfies a type of variational convergence to the original probability function. Consequently, when considering an appropriately structured problem involving probabilistic constraints, we can thus entail the convergence of the minimizers of the regularized approximate problems to the minimizers of the original problem. Finally, we illustrate our results with examples and applications in the field of (nonsmooth) joint, semidefinite and probust chance constrained optimization problems.

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